Problem 1

(a). Explain why it might be valuable for market makers to observe the order flow, and why this might be bad for traders.

Solution: See the models on transparency from lecture 9 for an intuition for this question. Also, the Rudiger/Vigier paper deals with (potentially) informed market makers, although this information does not come from the order flow. Market makers who observe the order flow can learn about the private information of traders. This then in turn makes them better informed than other market makers, and gives them an 'information monopoly', which they can use to pick out good trades and leave the bad trades to less informed market makers. As a consequence, the less informed market makers (those who did not observe the previous order flow) will set a higher spread, to protect themselves against this adverse selection.

We saw in lecture 9 that this dynamic can lead market makers to bid more aggressively for early order flow (i.e. in the first period of the model we saw, the spread was lower than in a standard model), whereas they bid less aggressively for the later order flow due to the adverse selection. Thus, traders gain initially but lose later. In the model we analyzed, the net effect was a loss for the traders, the reason being that essentially the market makers who became informed became information monopolists, thereby disrupting the usual market maker competition.

(b). In limit order books, traders choose themselves whether to provide liquidity (post limit orders) or to take liquidity (post market orders). Thus, in some sense, traders choose between being market makers and speculators, in the language of the Glosten/Milgrom model. What might affect their choice of becoming one or the other?

Solution: As seen in the model of Parlour in lecture 6, this choice can depend on the private valuation of the trader. A trader with very high or very low private valuation may prefer a high execution probability and therefore choose a market order, whereas a trader who is more concerned with the common value may opt for lower execution probability in order to (potentially) get a better price.

Another reason for acting as a liquidity provider could be greater patience. If a limit order is not immediately executed, one always has the possibility of withdrawing it and posting a new one (although possibly at cost) and therefore traders with greater patience may prefer to accept a potentially longer waiting time until the execution of their order in exchange for a better price.

A third reason could be technology. We discussed in problem set 2 how high-frequency traders use a strategy known as 'front running' to make money by marking up limit orders. Thus, issues of speed also influence the choice of whether to provide or to take liquidity. A 'slow trader' who posts limit orders is potentially at a disadvantage to high-frequency traders who post limit orders. However, the same might be said of those traders who were victims of front-running, and therefore it is not necessarily clear whether this tips the balance in one direction or the other.

(c). Explain why it is that, given a set of limit orders, a uniform price auction will always be efficient in the sense that after the auction takes place, the sellers who did not sell must necessarily have posted a higher price than those sellers who did sell, and the buyers who did not buy must necessarily have posted a lower price than those buyers who did buy. Compare this with other market mechanisms (other auction types, continuous limit-order books, etc.). What may be an argument for not always prioritizing efficiency?

Solution: We talked about this at the end of lecture 1 and the beginning of lecture 2. The uniform price auction can be thought of as clearing of first the trades with the highest 'spreads', i.e. the the highest bids with the lowest asks. It continues this process until there are no more trades to clear off. Thus, at the clearing price, there are no more trades than can be carried out. Exactly because the orders are ranked according to the price they post, the orders with the best price will be executed first.

This is generally true in all auction settings, also discriminatory-price auctions and more fancy formats, since they usually prioritize the order-clearing according to price. However, it is not true in the continuous limit-order book. Here, orders are cleared continuously, and therefore an early order may be prioritized over a later order with a better price. In a dealer market, the issue is similar, since trades are normally carried out on a continuous basis.

Auctions have good efficiency properties, but in large part this is due to the fact that they aggregate orders. Thus, the efficiency of the auction depends very much on its frequency. The more frequent the auction becomes, the more it will resemble a continuous LOB in the sense that the timing of orders will matter a great deal. Therefore, if one wants to have an auction market with frequent trading, efficiency gains compared to other mechanisms may be small. And since speed is often a variable of interest to traders, many times it will be beneficial to sacrifice some efficiency in order to gain more frequent execution.

Another issue is that in an auction, it is normally not possible to see the bids of the other bidders before the auction, and therefore the eventual trading price and trading probability is in some sense much less transparent than in an LOB or dealer market, where the trader may have a very good idea about current prices from looking at the book or at the quotes. On the other hand, auctions are useful in situations where no consensus-price exists, since they allow for many parties to submit their bids and then to aggregate this information into a price.

Problem 2

In this question we use a Glosten-Milgrom style model to analyze information acquisition by market makers. The model has the following features:

- **Setup.** Consider the market for a risky asset with value V. For simplicity, $V = 1$ with probability $\frac{1}{2}$ and $V = 0$ with probability $\frac{1}{2}$.
- **Market makers.** There are two market makers (MMs, indexed by $n = 1, 2$). The MMs simultaneously set ask and bid prices a_n and b_n. If MMn gets to trade, his profit is $a_n - V$ if the incoming order is a buy order, and $V - b_n$ if the incoming order is a sell order. If he does not trade, his profit is zero.
- **Price priority.** There is price priority on the market, such that the MM with the best price always receives the incoming order. If the MMs set the same price, each gets the order with probability 1/2. Let the 'best prices' be denoted by $\hat{a} := \min_n a_n$ and $\hat{b} := \max_n b_n$.
- **Traders.** There is a single trader, who is *always a noise trader* and buys/sells one unit with probability 1/2.
- **Information acquisition.** Before trading takes place, each MM can learn V at a cost of $c > 0$. Neither the decision of whether to learn V nor the value of V is observed by the other MM. Let p_n be the probability that MMn acquires information.

We will look for a perfect Bayesian equilibrium (PBE). We now proceed to solve the model. Since there is only noise trading in the model and since everything is symmetric, we will focus throughout on the bid price b_n (the price at which the MM buys the asset, and the trader sells).

(a). Suppose we are analyzing an equilibrium in which the MMs' strategy is to **not** acquire information ($p_n = 0$ for $n = 1, 2$). Argue that in this case, $b_n = 1/2$ in equilibrium.

> **Solution:** When MMs are uninformed, the expected value of the asset is 1/2. Since there is only noise trading, the sign of the incoming order does not reveal any information. Hence, Bertrand competition between the two MMs drives the price to the expected value, that is, 1/2.

(b). Suppose MMn chooses never to become informed ($p_n = 0$), and MMm chooses always to become informed ($p_m = 1$). Suppose the uninformed MM sets prices $0 < b_n \leq a_n < 1$. Argue that in this case, the informed MM should optimally set bid (ask) price marginally above (below) the informed MM's bid (ask) price whenever $V = 1$ ($V = 0$). As for the ask (bid) price when $V = 1$ ($V = 0$), any price such that the informed MM will not sell (buy) the asset is optimal.

> **Solution:** When $V = 1$, the informed MM would like to buy the asset at the lowest possible price, but at most at 1. Since $b_n < 1$, then setting b_m marginally above b_n achieves this. On the other hand, when $V = 1$, the informed MM cannot make money from selling the asset, since he can at best fetch a price $a_n < 1$. Therefore, any price $a_m > a_n$ (which will imply that he does not sell) is optimal for the informed MM.

(c). Show that the lowest c such that no MM becomes informed in equilibrium is $c = 1/4$. That is to say, MMs will be uninformed for $c > 1/4$, but informed ($p_n > 0$ for at least one MM) for $c < 1/4$.

> **Solution:** If no-one is informed, $a_n = b_n = 1/2$ as established above. The profit to MMn from deviating to become informed is the following. If $V = 1$, marginally set b_n above 1/2 so as to win any potential incoming sell-order. If the noise trader order is a sell order (happens with probability 1/2), will then make profit of $1 - 1/2$. If $V = 0$, marginally set a_n below 1/2 so as to win any incoming buy-order. If the noise trader order is a buy order (happens with probability 1/2), will then make profit of $1/2 - 0$. Thus, the expected profit from becoming informed is $1/2 \cdot 1/2 = 1/4$. The expected profit from not becoming informed is 0, since if there is trade, this will happen at price 1/2, which is exactly the expected value to an uninformed MM. Thus, MMs will be uninformed for $c > 1/4$, but informed ($p_n > 0$ for at least one MM) for $c < 1/4$.

(d). Suppose MM1 becomes informed for sure ($p_1 = 1$) whereas MM2 does not become informed ($p_2 = 0$). Calculate the bid price set by MM2 in this case (calculate b_2) and argue that $p_1 = 1$ and $p_2 = 0$ can never be an equilibrium.

Hint: Consider three cases: $b_2 < 0$, $b_2 = 0$, and $b_2 > 0$ to find the optimal price b_2. Then calculate the gains to being informed versus being uninformed for both MMs.

Solution: First, consider $b_2 > 0$. Notice that if $b_2 > 0$, then MM1 never buys when $V = 0$. Thus, the expected value of the asset for MM2 is always strictly less than 1. Therefore, setting $b_2 \geq 1$ can never be optimal. Furthermore, if MM2 knows that MM1 is informed for sure, then he also knows that whenever $b_2 \in (0, 1)$, MM2 will only be able to buy the asset if $V = 0$, and therefore $0 - b_2 < 0$ implies that this is not optimal. Second, consider $b_2 < 0$. This would imply that MM1 sets $b_1 \in (b_2, 0)$ even when $V = 0$, since this would be profitable for MM1. Thus, MM2 never trades and has zero profits, but would have strictly positive expected profits from setting $b_2 \in (b_1, 0)$. In conclusion, the only possible equilibrium price for MM2 is $b_2 = 0$. This will yield zero profits, since MM2 will only get the order when $V = 0$.

Now consider MM1. If MM2 sets $b_2 = 0$, this means that MM1's profits whenever $V = 1$ are $1/2 \cdot (1 - 0) = 1/2$ (probability of noise sell order times profit from buying at price 0). The ask-side is symmetric, so MM1's expected return from being informed is 1/2. But, by deviating and not being informed, MM1 could still set b_1 marginally above 0, and earn $1/2 \cdot (1/2 - 0) = 1/4$. Thus, MM1's gain from being informed is only 1/4.

On the other hand, if MM2 deviates and becomes informed, he could set b_2 marginally above b_1 (which is marginally above 0), and earn $1/2 \cdot (1 - 0) = 1/2$. When he is uninformed he earns zero (established above). So his gain from being informed is 1/2. Thus, it cannot be an equilibrium that MM2 is uninformed when MM1 is informed for sure.

(e). The previous question shows that it is not an equilibrium for one MM to be informed and the other not. Now, suppose both MMs become informed with the same interior probability:

$$p_1 = p_2 = p \in (0, 1).$$

Focus again on the bid side. When both MMs are potentially informed, the price-setting will be in mixed strategies. Suppose that if MMn is uninformed he uses the strategy

$$\sigma(b) = \mathbb{P}(b_n \leq b | \text{uninformed}),$$

and if MMn is informed and learns that the value is high ($V = 1$) he follows the strategy

$$\overline{\sigma}(b) = \mathbb{P}(b_n \leq b | \text{informed}, V = 1).$$

If MMn is informed and learns that the value is low ($V = 0$), he just sets $b_n = 0$ with probability one. Thus, the two MMs follow the same strategy. Notice that the strategies σ and $\overline{\sigma}$ indicate the probability that the MM sets a bid price below b.

Furthermore, suppose that there exist l and u with $0 < l < u < 1$ such that the following holds:

$$\sigma(0) = 0, \qquad \sigma(l) = 1, \qquad \overline{\sigma}(l) = 0, \qquad \overline{\sigma}(u) = 1.$$

Hence, the uninformed MM bids in the interval $[0, l]$, and the informed MM who learns that the value is high bids in the interval $[l, u]$.

Consider the gross profit (not counting potential information cost) from bidding b when a sell order arrives. Denote this $\Pi(b)$ for an uninformed MM and $\overline{\Pi}(b)$ for informed MM who observes $V = 1$. Show that if the MMs use the above strategy then:

$$\Pi(b) = \frac{1}{2}\left[p + (1-p)\sigma(b)\right](0 - b) + \frac{1}{2}[(1-p)\sigma(b)](1 - b);$$
$$\overline{\Pi}(b) = [p\overline{\sigma}(b) + (1-p)](1 - b).$$

Hint: For the uninformed MM, there are two possibilities: $V = 0$ and $V = 1$. Each possibility carries a different probability of winning. On the other hand, the informed MM who observed $V = 1$ knows the asset value.

Solution: For the uninformed type, the first term is the probability that the value is low times the probability of the other MM's bid price being below b in this case, times the profit from the trade conditional on the value being low. The second term is the probability that the value is high, times the probability of the other MM's bid price being below b in this case, times the profit from the trade conditional on the value being high.

For the informed type who knows that the value is high, the profit is the probability that the other MM's bid price is below b, times the profit from the trade conditional on the value being high.

(f). Argue informally that the uninformed MM must make zero profits: $\Pi(b) = 0$ for all $b \in [0, l]$. Then, together with $\sigma(l) = 1$, use this to find l.

Solution: The uninformed MM must make zero profit since he has no information to leverage – therefore, he will always be outbid by an informed MM. The best he can do is to protect himself against this potential adverse selection by setting a spread, but he cannot (in expectation) turn a positive profit.

Now, using the indication in the question, we calculate

$$\begin{aligned}\Pi(l) &= \frac{1}{2}\left[p + (1-p)\sigma(l)\right](0 - l) + \frac{1}{2}[(1-p)\sigma(l)](1-l) \\ &= \frac{1}{2}\left[-l + (1-p)(1-l)\right].\end{aligned}$$

Using $\Pi(l) = 0$ we thus obtain $l = \frac{1-p}{2-p}$.

(g). Assume that the informed MM will make a positive profit. Since he follows a mixed strategy, we must have $\overline{\Pi}(b) = \overline{\Pi} > 0$ for all $b \in [l, u]$. Use this together with $\overline{\sigma}(l) = 0$ and your answer to the previous question to find $\overline{\Pi}$.

Solution: We carry out the following derivation:

$$\begin{aligned}
\overline{\Pi} &= \overline{\Pi}(l) \\
&= [p\overline{\sigma}(l) + (1-p)](1-l) \\
&= [0 + (1-p)](1 - \frac{1-p}{2-p}) \\
&= [0 + (1-p)](1 - \frac{1-p}{2-p}) \\
&= \frac{1-p}{2-p}.
\end{aligned}$$

(h). Since everything is symmetric in the model, the expected gross profit to an informed MM is the probability of a sell order times the gross profit conditional on a sell order, that is, it is $\frac{1}{2} \cdot \overline{\Pi}$. For $c \in (0, 1/4)$, we will have $p \in (0, 1)$. Since MMs use a mixed strategy in information acquisition, their net profits from acquiring information (gross profit less c) must be zero. Use this to find the equilibrium probability $p^*(c)$ of acquiring information as a function of information cost.

Solution: Using the information given in the question we have,

$$\begin{aligned}
\frac{1}{2} \cdot \overline{\Pi} - c = 0 \\
\frac{1}{2} \cdot \frac{1-p}{2-p} - c = 0 \\
p^*(c) = \frac{1-4c}{1-2c}.
\end{aligned}$$

Problem 3

A great deal of the course has been devoted to analyzing how adverse selection drives pricing in financial markets. We have mainly assumed that only market makers faced adverse selection (for instance, the models by Glosten/Milgrom and Kyle), but we have also touched on the possibility that market makers can be informed. **Discuss, using real life examples, how adverse selection (on both sides of the market) may affect market outcomes.**

You should not limit yourself only to the arguments we have seen in class, but think more broadly about the question. You may wish, but are not obliged, to consider the following issues: (i) The distinction between market makers and traders is not always clear, e.g. in a limit order book you can choose between providing liquidity or taking it, you are not bound to a role a priori. (ii) Market makers are most often large institutions, traders may be large or small. (iii) Regulation on information sharing between different branches of large banks (so-called 'Chinese walls'). (iv) The role of high-frequency trading in market making/speculation and how this affects information-driven adverse selection.

Solution: Individual.